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Modelling swimmers' speeds over the course of a race

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Abstract

A stochastic model of swimming speed over the course of a male 200m freestyle swimming race is proposed. It builds on a dynamical model reflecting the trade-off between drag and propulsion in swimming. The parameters of the model are estimated from elapsed time data observed at several points along the pool. The model fits the data well and also provides a good description of the swimming strategies of each swimmer from phase to phase in the race. An individual factor measuring how much faster or slower the individual swims relative to the average swimming speed of the race is simultaneously estimated in the course of fitting the model. This factor is, as expected, closely related to the final outcome of the race.

Key words: Drag, Freestyle, Modelling, Propulsion, Swimming speed.

1 Introduction

There have been many attempts to model aspects of swimming from various points of view including Biomechanics, Physiology and Race Analysis. However, there appear to have been few that model swimming speed over the course of a race. In this paper, we propose a new model of swimming speed and its variation over the race, where this model is fitted to elapsed times at several points along the side of a pool. The model provides a good description of the strategies adopted by each swimmer over the course of the race. As a consequence, it should be of use to trainers, national selectors and those interested in the biomechanics of swimming.

The observations used here are elapsed times observed at 21 check points in the 34 preliminary male 200m freestyle race held in the 2004 Japan Swimming Championships. A suitable dynamical model is fitted that includes a parameter describing the individual effect of each swimmer. Since a swimmer's strategy may change from location to location in a lap, each lap is split into 3 phases, the first, middle and last. We make use of similarities between phases over laps, although the first and last laps need special attention, leading to a more parsimonious model with reduced number of parameters. This is an important consideration here since the number of observations is limited. Section 3 demonstrates how to accomplish this task.

Our proposed model builds directly on the deterministic models of Amar (1920), Karpovich (1933), Kolmogorov and Duplishcheva (1992) and Takagi *et al.* (1999). It extends these models by accounting for propulsion and setting them in a suitable stochastic framework. Related work from the view point of race analysis includes Arellano *et al.* (1994), Chengalur and Brown (1992), Craig and Pendergast (1979), Craig *et al.* (1985), Ikuta *et al.* (1998), Kjendlie *et al.* (2004), Matsui *et al.* (1997) and Okuno *et al.* (2003). These papers focus mainly on the swimming speed in the middle phase, which is decomposed into a product of the stroke length (m/cycle) and stroke ratio (cycle/min). Relations between these two factors are mainly discussed from a largely empirical point of view. It would be natural to concentrate on such aspects if swimming in the middle phase of the race were the key to winning. However, our model shows that swimming strategies in other phases are equally important for a good outcome.

Our approach is to model all phases of the race to allow a better understanding of individual strategies for each phase and their impact on the race as a whole. In this way, an overall integrated strategy for improvement of swimming performance can be developed for the entire race.

2 Data

The 2004 Japan Swimming Championships was not only one of the major swimming competitions in Japan, but also part of the selection procedure for the Athens Olympic games. For the male 200m freestyle race, only 34 qualified swimmers holding a record faster than 1:50.8 were invited . The race was recorded on video tapes by the Medicine and Scientific Committee of Japan Swimming Federation, with the aim of using them for scientific research. The purpose and the design of the video recording were clearly explained by the committee to team managers prior to the race and they gave their informed consent. The authors are allowed to use these video tapes from the committee with the proviso that the privacy and dignity of the swimmers should be protected.

The race was recorded on video tapes by 5 video cameras (60 frames per second) placed parallel to the swimming direction. To minimise perspective bias, each camera focused on just one of the intervals: 5–7.5, 10–15, 20–30, 35–40, 42.5–45m. Based on the time stamp which was accurate to within 5 milliseconds on each frame, elapsed times were measured when a swimmer's head reached each one of 21 check points: 0, 15, 20, 30, 45, 50, 57.5, 70, 80, 95, 100, 107.5, 120, 130, 145, 150, 157.5, 170, 180, 195 and 200m, with the exception of the ends of the pool where elapsed times were measured when a swimmer touched the wall. The second check point in the first lap was placed at 15m instead of 7.5m since it is hard to identify the location of each swimmer for 15m after a dive. More details of the data collection can be found in Matsui *et al.* (1997).

3 Model

3.1 An underpinning deterministic model

A well known model for swimming speed $v(t)$ at time t is given by the differential equation,

$$\frac{dv(t)}{dt} = -\alpha v(t)^2,$$

which was proposed by Amar (1920). There have been several experiments to estimate the value of the drag parameter $\alpha > 0$ in relation to the body characteristics of each swimmer (Karpovich 1933, Kolmogorov and Duplishcheva 1992, Takagi *et al.* 1999). However, this model ignores the effect of propulsion which needs to be taken into account in swimming. A more general model is

$$\frac{dv(t)}{dt} = -\alpha v(t)^2 + \beta, \tag{1}$$

where $\beta \geq 0$ measures the propulsion generated by the swimmer's stroke action. The solution of the differential equation (1) can be explicitly written as

$$v(t) = \begin{cases} \dfrac{1}{\alpha t + \frac{1}{v_0}} & (\beta = 0) \\ \\ \dfrac{2\sqrt{\kappa}}{1 - c_1 e^{-2\alpha\sqrt{\kappa}t}} - \sqrt{\kappa} & (\beta > 0) \end{cases} \qquad (t \geq 0),$$

where v_0 is the initial speed, $\kappa = \beta/\alpha$ and $c_1 = (v_0 - \sqrt{\kappa}) / (v_0 + \sqrt{\kappa})$. The model is continuous in terms of β so that

$$\lim_{\beta \to +0} v(t) = \frac{1}{\alpha t + \frac{1}{v_0}}.$$

It is better to write the speed as a function of distance x rather than time t since our observed elapsed times are measured in terms of distance. As is shown in Appendix A, the swimming speed $v(x)$ at distance x is given by

$$v(x) = \begin{cases} v_0 e^{-\alpha(x - x_0)} & (\beta = 0) \\ \\ \sqrt{c_2 e^{-2\alpha(x - x_0)} + \kappa} & (\beta > 0) \end{cases} \qquad (x \geq x_0), \tag{2}$$

provided that the speed at the initial distance x_0 is v_0 and $c_2 = {v_0}^2 - \kappa$. However it is not appropriate to apply this model to the whole race directly since the male 200m freestyle race consists of 4 laps of the pool, each of length 50m. It is clear that α or β will not stay constant over the race, nor even in a lap so a natural approach is to split each lap into several phases within which these parameters might be expected to be constant.

For simplicity, we introduce 3 phases in each lap. The first phase (from 0m to x_1m) is just after a dive or turn where drag, but no stroke propulsion, are expected ($\alpha > 0, \beta = 0$). By contrast, drag and propulsion are both expected ($\alpha > 0$, $\beta > 0$) in the middle phase (from x_1m to x_2m) and in the last phase (from x_2m to 50m). It is also natural to assume that a swimmer's speed stays constant in the middle phase since every swimmer should have reached an equilibrium swimming state in this phase, so that $v(x_1) = v(x) = v(x_2)$ for $x_1 \leq x \leq x_2$, that is, $\kappa = {v_0}^2$. Such an equilibrium no longer holds true in the last phase where a swimmer should have prepared for a turn or the end of the race. Also, note that the drag parameter in the first phase can be different from that in other phases because of the dive or turn in the first phase.

Combining these assumptions, a model for swimming speed of a lap which consists of 3 phases is then

$$v(x;\boldsymbol{\theta}) = \begin{cases} v_0 e^{-\alpha_0 x} & (0 \leq x < x_1), & \text{(First phase)} \\ v(x_1) & (x_1 \leq x < x_2), & \text{(Middle phase)} \\ \sqrt{c_2 e^{-2\alpha(x-x_2)} + \kappa} & (x_2 \leq x < 50), & \text{(Last phase)} \end{cases}$$

where $\boldsymbol{\theta} = (v_0, \alpha_0, x_1, x_2, \alpha, \beta)$, $c_2 = v(x_1)^2 - \kappa$ and $\kappa = \beta/\alpha$. Note that the break points x_1 and x_2 are also parameters which can differ from lap to lap. Figure 1 shows a stylised picture of the swimming speed $v(x)$. Then the swimming speed over the whole race is given as

$$v_j(x) = v(x - 50(j-1); \boldsymbol{\theta}_j),$$

for $50(j-1) \leq x < 50j$, $j = 1, 2, 3, 4$, where $\boldsymbol{\theta}_j = (v_{0j}, \alpha_{0j}, x_{1j}, x_{2j}, \alpha_j, \beta_j)$ is the vector of parameters for lap j. Therefore the set of parameters $\{\boldsymbol{\theta}_j; j = 1, 2, 3, 4\}$ determines a swimming speed model over the race. The estimation of such unknown parameters will be discussed in Section 3.2.

An important aspect of the model is the specification of an individual effect for each swimmer. We model the swimming speed of swimmer i in lap j as

$$\mu_i v_j(x), \ j = 1, 2, 3, 4,$$

where μ_i is a multiplicative factor, specific to the individual swimmer, that is assumed to be constant over the race, and $v_j(x)$ is the common swimming speed of swimmers in lap j. This multiplicative model allows for a simple understanding of a swimmer's performance relative to the common swimming speed $v_j(x)$. In particular, the values of the multiplicative factors μ_i provide an overall measure of swimming performance that can be used to discriminate between swimmers.

3.2 A stochastic model for swimming speed

The observed elapsed times are not free from random fluctuations due to the swimmers as well as random errors in the observational process. If $T_{ij}(k)$ denotes the elapsed

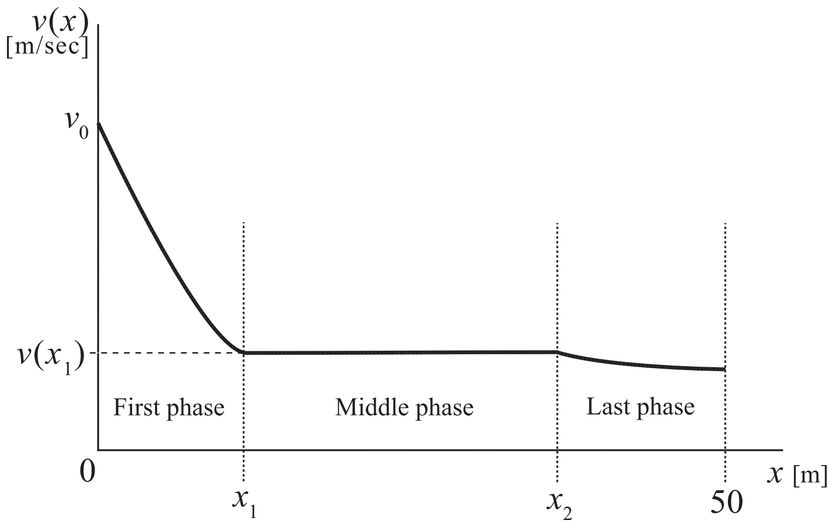

Figure 1: A stylised picture of swimming speed $v(x)$ over one lap of the race. The lap is divided into 3 phases, a first phase just after a dive or turn, a middle phase and a last phase just before a turn or the finish of the race.

time of swimmer i at distance $x_j(k)$, where k denotes a check point in lap j, we assume that

$$T_{ij}(k) = \int_0^{x_j(k)} \frac{1}{\mu_i v_j(x)}\, dx + \sigma B_i(x_j(k)), \tag{3}$$

where $\{B_i(x); 0 \le x < 200\}$ is standard Brownian motion representing accumulated error up to distance $x_j(k)$. Brownian motion is a continuous time process, which is widely used in various disciplines. Its basic property is that any increment $B_i(x+\Delta x) - B_i(x)$ is normal with mean zero and variance Δx and distributed independently of any other non-overlapping increment.

Thus

$$\Delta T_{ij}(k) = \frac{1}{\mu_i} \int_{x_j(k-1)}^{x_j(k)} \frac{1}{v_j(x)}\, dx + \sigma \sqrt{\Delta x_j(k)}\, \varepsilon_{ijk},$$

where $\Delta T_{ij}(k) = T_{ij}(k) - T_{ij}(k-1)$, $\Delta x_j(k) = x_j(k) - x_j(k-1)$ and the ε_{ijk} are independent standard normal random variables. The parameters of the model can now be estimated by weighted least squares

$$\sum_{i=1}^{34} \sum_{j=1}^{4} \sum_{k=1}^{5} \frac{r_{ijk}^2}{\Delta x_j(k)},$$

where

$$r_{ijk} = \Delta T_{ij}(k) - \frac{1}{\mu_i} \int_{x_j(k-1)}^{x_j(k)} \frac{1}{v_j(x)}\, dx.$$

The squared residual r_{ijk}^2 is divided by $\Delta x_j(k)$ because the residuals $\{r_{ijk}\}$ are expected to be independently distributed normal random variables with mean zero and variance $\sigma^2 \Delta x_j(k)$. The normality will be checked in Section 4.3.

To keep the model parsimonious we now reduce the number of parameters $\{\boldsymbol{\theta}_j; j = 1, 2, 3, 4\}$ by assuming similarities between phases over laps. The model for the first phase is assumed to be common over laps other than the first phase in the first lap since this starts from a dive. We also assume that the parameters α_{0j} and x_{1j} are common over the laps other than the first ($\alpha_{02} = \alpha_{03} = \alpha_{04}$ and $x_{12} = x_{13} = x_{14}$), and that the x_{2j} are common over laps other than the last ($x_{21} = x_{22} = x_{23}$). The break point in the last lap x_{24} is different since all swimmers are focused on completing the race rather than making a turn. Furthermore, we assume that the drag parameter α_j for the last phase in each lap is assumed to be known. This is needed for stable estimates. We will use $\alpha_j = 0.428$ or $\alpha_j = 0.37$ for any lap j, that are the constants known from the results of an experiment by Toussaint *et al.* (1988) and Karpovich (1933) for a 70 kg swimmer. It will be seen that the choice of either of these values does not lead to any significant difference in the final results. These considerations reduce the total number of parameters to be estimated to 48 for the whole race.

Fortunately, we can apply the above parameter estimation procedure without any numerical integration. The integration of $1/v(x)$ for the swimming speed given in (2) is explicitly written as

$$\int_0^x \frac{1}{v(u)}\, du = \begin{cases} \dfrac{1}{\alpha}\left(\dfrac{1}{v(x)} - \dfrac{1}{v_0}\right) & (\beta = 0)\,, \\[2ex] \dfrac{1}{2\alpha\sqrt{\kappa}}\left\{\log\left(\dfrac{v(x)+\sqrt{\kappa}}{v(x)-\sqrt{\kappa}}\right) - \log\left(\dfrac{v_0+\sqrt{\kappa}}{v_0-\sqrt{\kappa}}\right)\right\} & (\beta > 0)\,. \end{cases} \tag{4}$$

The proof is given in Appendix B. To estimate the parameters, a program for solving nonlinear least squares, `nlminb` in S-PLUS (Chambers and Hastie 1992), was employed.

4 Result

4.1 Common swimming speed and its parameters

The estimated parameters assuming $\alpha_j = 0.428$ and $\alpha_j = 0.37$ for the last phase of each lap are listed in Table 1 and Table 2 respectively. Note that the choice has little effect on the parameter estimation, as expected.

The estimated parameters in Table 1 and Table 2 are largely consistent with the experience of swimmers and their trainers. They also provide a good description of the characteristics of swimming in a race. Diving not only affects the initial speed $\hat{v}_{0j}$ but also the drag parameter $\hat{\alpha}_j$ and the location parameter $\hat{x}_{1j}$. As expected, $\hat{v}_{01}$ and $\hat{x}_{11}$ in the first lap are higher than the values in other laps. We can also see how swimmers exhaust their energy as the race progresses with the initial speed $\hat{v}_{0j}$ in each lap decreasing by approximately 0.07 m/s per lap. A reason why the drag parameter $\hat{\alpha}_{01}$ in the first lap is higher than other laps could be due to the impact of diving.

The effect of the finish line is also apparent with the values of $\hat{x}_{24}$ and $\hat{\beta}_4$ in the last lap being higher than those in the other laps. Since no turn is necessary at the end of the last lap, each swimmer makes their break for the finish line over the last phase of the race.

Table 1: Estimated parameters ($\alpha_j = 0.428$).

<table>
<tr><th>Lap</th><th>$\hat{v}_{0j}$</th><th>$\hat{\alpha}_{0j}$</th><th>$\hat{x}_{1j}$</th><th>$\hat{x}_{2j}$</th><th>$\hat{\beta}_j$</th></tr>
<tr><td>$j = 1$</td><td>4.11</td><td>0.09</td><td>9.32</td><td rowspan="3">36.74</td><td>1.17</td></tr>
<tr><td>$j = 2$</td><td>3.06</td><td rowspan="3">0.08</td><td rowspan="3">7.05</td><td>1.09</td></tr>
<tr><td>$j = 3$</td><td>3.00</td><td>1.04</td></tr>
<tr><td>$j = 4$</td><td>2.93</td><td>45.00</td><td>1.42</td></tr>
</table>

Table 2: Estimated parameters ($\alpha_j = 0.37$).

<table>
<tr><th>Lap</th><th>$\hat{v}_{0j}$</th><th>$\hat{\alpha}_{0j}$</th><th>$\hat{x}_{1j}$</th><th>$\hat{x}_{2j}$</th><th>$\hat{\beta}_j$</th></tr>
<tr><td>$j = 1$</td><td>4.10</td><td>0.09</td><td>9.32</td><td rowspan="3">36.55</td><td>1.16</td></tr>
<tr><td>$j = 2$</td><td>3.06</td><td rowspan="3">0.08</td><td rowspan="3">7.05</td><td>1.09</td></tr>
<tr><td>$j = 3$</td><td>3.00</td><td>1.04</td></tr>
<tr><td>$j = 4$</td><td>2.92</td><td>45.00</td><td>1.43</td></tr>
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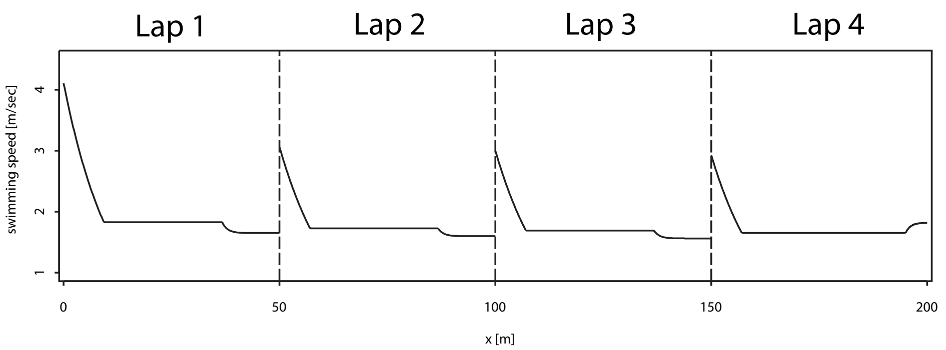

Figure 2: Estimated common swimming speed $\hat{v}_j(x)$ over the course of the race as a function of distance x and lap j ($j = 1, 2, 3, 4$).

Figure 2 illustrates the estimated common swimming speed. The speeds in the middle phase in each lap are 1.83, 1.73 1.67 and 1.65 m/sec respectively. These values are consistent with the values reported by Matsui *et al.* (1997) and Ikuta *et al.* (1999). Any unnatural behaviour of the common swimming speed, especially around the break points, is most likely due to the assumptions made for the parsimonious parameterisation discussed in the previous section. Such behaviour could be improved if more check points were set and more data collected, particularly around phase boundaries.

4.2 Individual parameters

Individual effects are measured by parameter $\mu_i, i = 1, 2, \ldots, 34$. Figure 3 shows that, as expected, the estimated μ_i are strongly and inversely related to the final time taken to complete the race. The reason why the $\hat{\mu}_i$ are not exactly placed on the theoretical line is not only because of estimation and measurement error, but also because of the random fluctuations of effort by each swimmer in the race. The point in the top left corner of the plot corresponds to the winner of the race. The isolation of this point from the others suggests that the winner is significantly faster than the others, with his individual factor being more than 3% faster than the averaged swimming speed. By contrast, the point in the bottom right of the plot corresponds to the slowest swimmer whose factor was about 2% slower. As mentioned previously, these factors are important as they discriminate between the swimmers.

4.3 Discussion

Figure 4 plots the standardised residuals,

$$\left\{ \hat{\varepsilon}_{ijk} = \frac{\hat{r}_{ijk}}{\hat{\sigma}\sqrt{\Delta x_j\left(k\right)}}; i = 1, 2, \ldots, 34 \right\}$$

for every lap j and check point k. The dashed horizontal lines placed at ± 3.03 indicate the 95% confidence bound for the standardised residuals of each swimmer. The bound $b = 3.03$ is calculated so that

$$\mathrm{P}\left(|E_{jk}| < b, j = 1, 2, 3, 4, k = 1, 2, \ldots, 5\right) = 0.95,$$

where the $\{E_{jk}\}$ are independent standard normal random variables. In fact, b is the solution of $1 - (1-p)^{20} = 0.05/2$ where $b = \Phi^{-1}(1-p)$ and $\Phi(\cdot)$ is the standard normal distribution function.

Three standardised residuals lie outside the 95% confidence bounds. These are for swimmers ranked 13, 28 and 31 whose residual plots are shown in Figure 4. Residual plots such as these should be of use to swimmers and their trainers to evaluate their performance and the strategy they have adopted in a race. For example, the plot of the swimmer ranked 13 suggests that his rank would improve if he swam faster in the first and last lap. The swimmer ranked 28 has residuals that take high values before making a turn which implies a need to improve his turn technique. It is clear that the

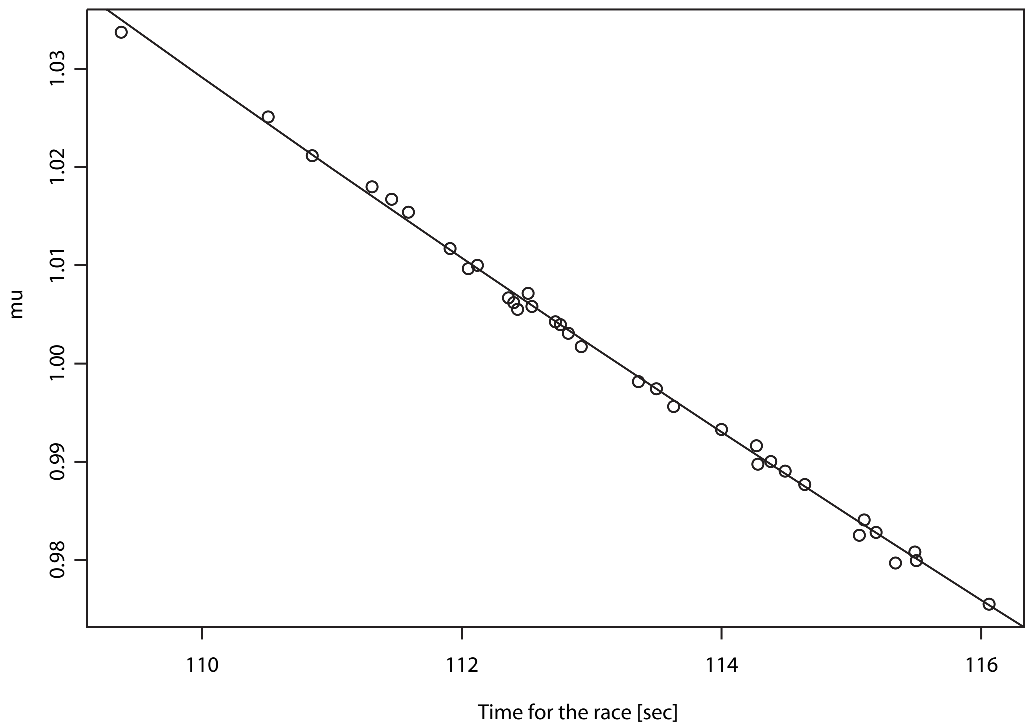

Figure 3: Individual fitted parameters $\hat{\mu}_i$ plotted as a function of race times together with the theoretical relationship.

swimmer ranked 31 started well, but exhausted his energy before the finish and failed to keep up with the other swimmers in the last lap. No doubt there are other factors that affect swimming performance (e.g. health, fitness, strategy etc) and these could be accounted for, but are left for future investigation.

With the exception of the 3 outlying swimmers mentioned above, the normal Q-Q plot (normal quantile-quantile plot; Chambers *et al.* 1983) of the standardised residuals $\hat{\varepsilon}_{ijk}$ was highly linear which supports the normality of the ε_{ijk}. This indicates that the parsimonious model adopted is a good fit to the data.

5 Conclusions

A stochastic model of swimming speed over the entire course of a race has been developed. It builds on a deterministic physical model that reflects the trade-off between drag and propulsion in swimming. The model has been simplified to cope with the limited number of observations, noting similarities and dissimilarities between the 4 laps of the race where each lap is divided into 3 separate phases. The elapsed times that are observed are modelled as a function of a deterministic function of distance swum, lap of the race and phase of the lap, together with accumulated stochastic error which is modelled using Brownian motion.

The model fits the data well, is easy to understand and interpret, and also provides a good description of the swimming strategies of each swimmer from phase to phase in

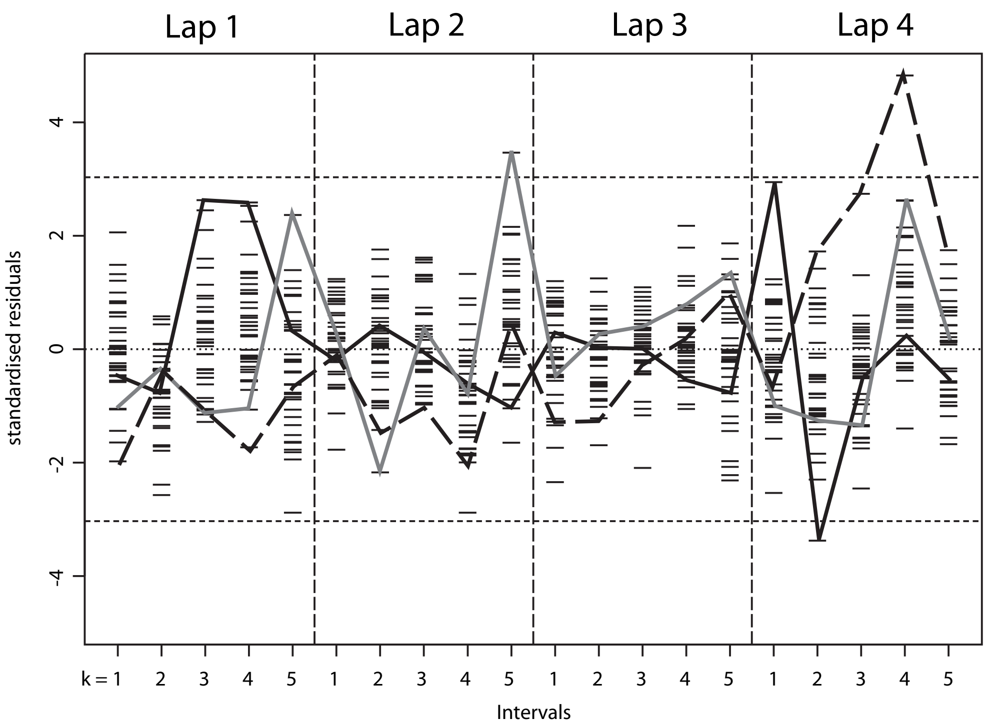

Figure 4: Standardised residuals with 95% confidence bounds. The residual plots of swimmers ranked 13 (solid black line), 28 (solid grey line) and 31 (dashed black line) are superimposed.

the race and over the race as a whole. An individual factor measuring how much faster or slower an individual swimmer performs relative to the average swimming speed of the race is simultaneously estimated in the course of fitting the model. This factor is, as expected, closely related to the final outcome of the race.

The model can be used to analyse and quantitatively evaluate the performance of individual swimmers. As a consequence, it should be of use to trainers and national selectors to improve individual swimming performances and to identify a swimmer's future potential. The model is also intended to be of interest to engineers and scientists concerned with the biomechanics of swimming and we hope that it will lead to a number of further developments.

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Appendix A Swimming speed as a function of x

To derive $v(x)$ from $v(t)$, it is enough to consider the case when $\beta > 0$. From the relation between the time and the speed,

$$t = \frac{1}{2\alpha\sqrt{\kappa}} \log\left(c_1 \frac{v(t) + \sqrt{\kappa}}{v(t) - \sqrt{\kappa}}\right),$$

we have

$$\begin{aligned} x(t) &= x_0 + \int_0^t v(s)\ ds \\ &= x_0 + 2\sqrt{\kappa}\int_0^t \left(\frac{1}{1 - c_1 e^{-2\alpha\sqrt{\kappa}s}} - \frac{1}{2}\right)\ ds \\ &= x_0 + \sqrt{\kappa}t + \frac{1}{\alpha}\log\left(\frac{1 - c_1 e^{-2\alpha\sqrt{\kappa}t}}{1 - c_1}\right). \end{aligned}$$

This yields the desired result.

Appendix B Standard elapsed time

It is enough to prove (4) only when $\beta > 0$. Letting $v_0 = v(0)$, we have

$$\begin{aligned}\int_0^x \frac{1}{v(u)}\,du &= \int_{v_0}^{v(x)} \frac{1}{v}\frac{du}{dv}\,dv \\ &= -\int_{v_0}^{v(x)} \frac{1}{\alpha v^2 - \beta}\,dv \\ &= \frac{1}{2\alpha\sqrt{\kappa}}\left\{\log\left(\frac{v(x)+\sqrt{\kappa}}{v(x)-\sqrt{\kappa}}\right) - \log\left(\frac{v_0+\sqrt{\kappa}}{v_0-\sqrt{\kappa}}\right)\right\}.\end{aligned}$$

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